Experimental Verification of the Mach-Number Field in a Supersonic Ludwieg Tube

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1 Introduction

A new supersonic Ludwieg Tube was completed at Caltech in 2000. A Ludwieg Tube, first proposed by Hubert Ludwieg in 1955 in response to a competition (see also Ludwieg *et al.* [1]), is a blow-down wind tunnel in which the high-pressure reservoir is a long tube. The flow is started by rupturing a diaphragm. The advantage of the long high-pressure reservoir is that, during the time it takes for the expansion wave generated by diaphragm rupture to propagate to the far end of the tube and for its reflection to come back to the diaphragm station, a perfectly uniform reservoir devoid of wave reflections is available to drive a supersonic nozzle. In the Caltech Ludwieg Tube (LT) the diaphragm station is located downstream of the nozzle to permit a smooth transition from the tube to the nozzle. Also, the boundary layer developing in the long tube is sucked off through an annular throat just upstream of the nozzle entrance, so that a clean flow is generated. The 300 mm diameter tube is 17 m long, the nozzle was designed for a Mach number of 2.3 by John J. Korte of NASA Langley RC, and the test section cross section is 200×200 mm. The useful test time is 75 ms. The aim of the present work was to determine the detailed Mach-number and flow direction fields by using the method of visualization of Mach waves, that was first used by Meyer[2].

2 The Mach-wave method

In a steady supersonic flow, disturbances that cause flow deflections generate shock waves or expansion waves. If the disturbance is infinitesimally small, the wave generated makes an angle μ to the flow that is given by

$$\sin \mu = \frac{1}{M}, \tag{1}$$

where M is the local Mach number. If two disturbances generate waves that intersect at a point in the flow field, the local streamline direction bisects the angle made by the two waves, and the latter is therefore 2μ. If sufficiently weak waves can be generated in an experimental flow, so that their intersection angle approximates this value closely, visualization of the waves can be used to determine both the local flow direction and Mach number.

The method is particularly useful in plane flow, because weak plane waves are very easily detectable by line-of-sight-integrating methods such as schlieren or shadowgraph techniques. The two-dimensional nozzle of the LT is thus an ideal candidate for the method. The sensitivity of the method can be obtained by differentiating equation 1:

$$\frac{d\mu}{dM} = -\frac{1}{M\sqrt{M^2-1}}, \tag{2}$$

showing that it is infinitely sensitive at $M = 1$ and has zero sensitivity at $M = \infty$. At $M = 2.3$ a change of Mach number by 0.01 causes a change of μ of $0.12°$. The accuracy of the method depends on how weak detectable waves can be made, and how accurately the wave angles can be measured. In the Mach number range of the flow studied here (≤ 2.3) the numbers look promising.

In order to get a rough idea of how small the strength of the wave has to be for the wave angle to be close enough to μ, an inviscid computation of supersonic flow over a disturbance like a strip of adhesive tape was made. The result of this computation is shown in Fig 1. The calculation shows the characteristic divergence of the leading and trailing shock waves. According to weak shock theory (see Whitham[3]) these two shocks lie on a curve of the form

$$y = \frac{x}{\sqrt{M^2-1}} \pm a\sqrt{x}, \tag{3}$$

where a is a constant that depends on the obstacle shape. Thus, these two shocks give a very good measure of the Mach angle only in the very far field. In the physical flow, the disturbances are generated by strips of adhesive tape stuck to the top and bottom walls of the nozzle. They are only 0.1 mm thick, and are therefore embedded in the nozzle-wall boundary layer and see a flow at a Mach number much smaller than 2.3, so that the waves they generate are much weaker than those in the inviscid computation. This is the reason for our choice of the wedge-shaped leading edge in the computation. Two other features of the flow are the expansion fans from the leading and trailing shoulders of the obstacle. Two straight lines are drawn in the figure at the Mach angle, approximately from the two shoulders. As may be seen, the leading characteristic from the trailing shoulder is almost immediately parallel to the line.

For the computation, the software system Amrita, constructed by James Quirk[4], was used. Amrita is a system that automates and packages computational tasks in such a way that the

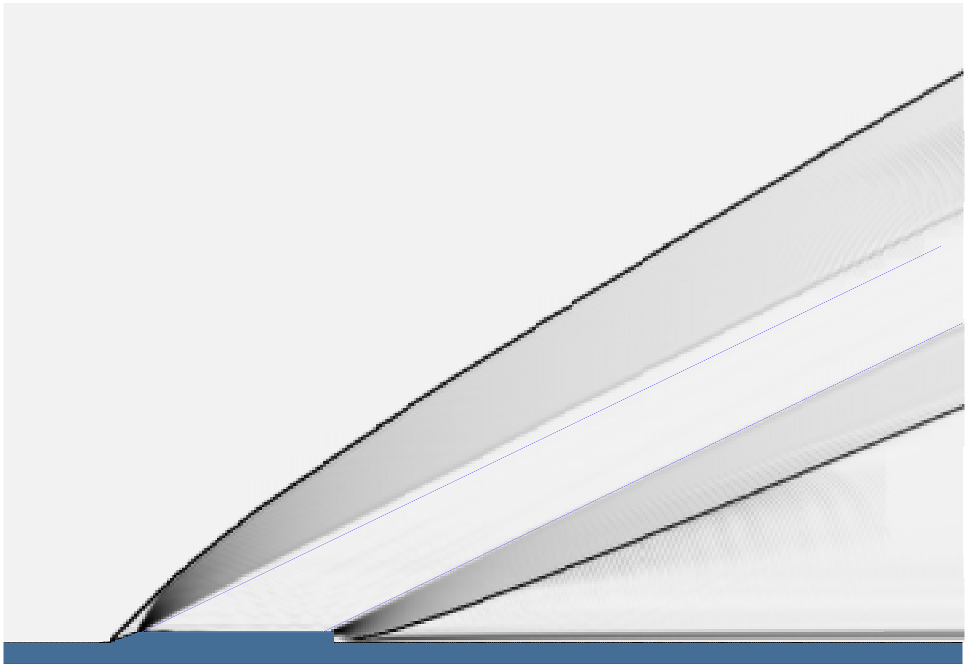

Figure 1. Pseudo-schlieren image of the computation of flow over a disturbance similar to a strip of adhesive tape. Note the divergence of the leading and trailing shocks. The two straight lines are drawn at the Mach angle, and it can be seen that the leading characteristic of the trailing expansion wave is almost immediately parallel to the straight line, while the other features are clearly still far from it at 40 disturbance thicknesses downstream.

packages can be combined (dynamically linked) according to instructions written in a high–level scripting language. The present application uses features of Amrita that include the automatic construction of an Euler solver, automatic adaptive mesh refinement according to simply chosen criteria, and scripting–language–driven computation and post–processing of the results. The Euler solver generated for the present computation was an operator–split scheme with HLLE flux and kappa-MUSCL reconstruction. The computation is made with a 90×60 coarse grid, to which, three levels of adaptive mesh refinement by a factor of three is applied, making the effective grid 810×540.

3 Results

The optical access to the test section and nozzle of the LT is almost complete, *i. e.*, the whole nozzle can be viewed from just downstream of the throat to the end of the test section, including the top and bottom boundary layer. This makes it particularly convenient to use the Mach wave method for measuring the Mach number distribution in the whole nozzle.

The disturbances are generated by placing equally spaced strips of adhesive tape at right angles to the flow on the top and bottom walls of the nozzle, see Fig. 2. The waves generated by the adhesive strips are then visualized using the schlieren technique. The schlieren system has an aperture of 250 mm, so that, even at the test section end of the nozzle, where the height of the flow is 200 mm, top and bottom walls can be seen in the same image. By moving the windows and the schlieren system, a composite view of the whole nozzle can be assembled from several runs of the LT. Such a composite schlieren picture of the nozzle flow is shown in Fig 3.

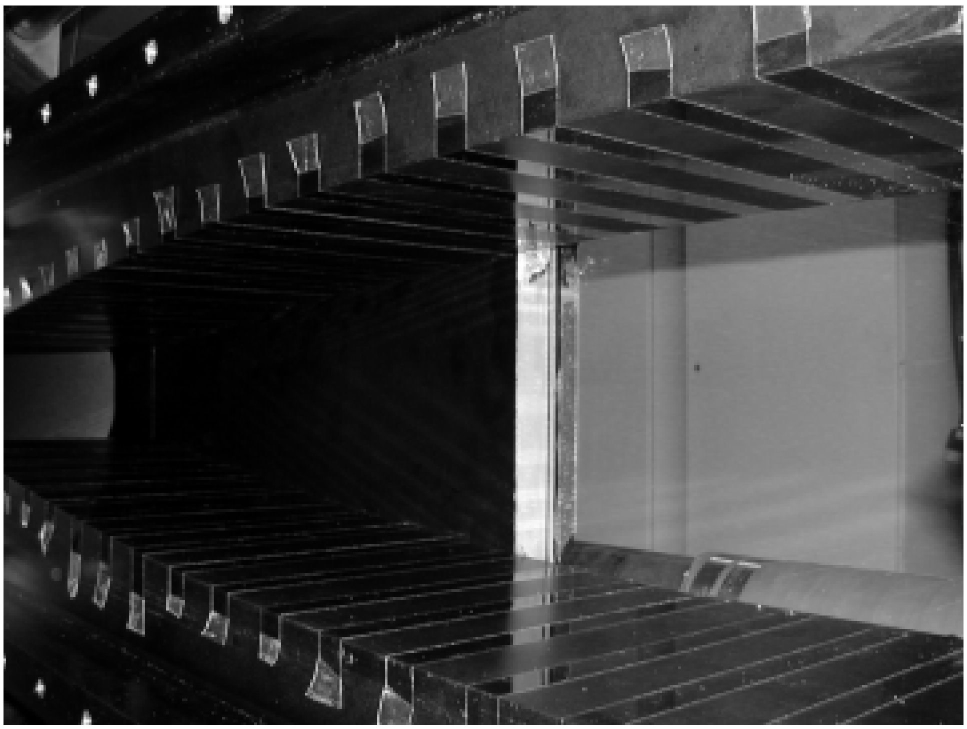

Figure 2. View of the nozzle with windows removed, showing the adhesive strips on the top and bottom nozzle walls.

As may be seen in this figure, the leading and trailing shocks off the adhesive tape diverge significantly at the upstream end of the nozzle, where the boundary layer is thin. A second reason for the divergence in the upstream region is that the axial Mach number gradient is quite large there. The divergence decreases rapidly further downstream as the boundary layer thickness increases and the Mach number gradient decreases. The boundary layer can be seen in parts of the picture. Note how, at the downstream end, the curvature of the Mach waves in the boundary layer can clearly be seen.

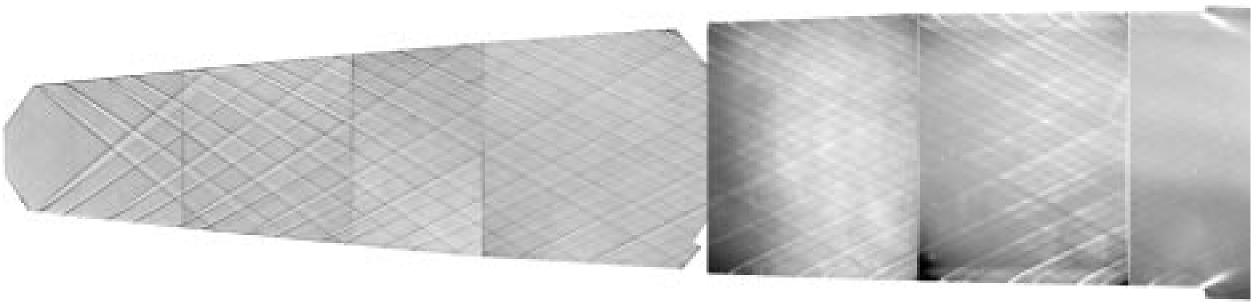

Figure 3. Composite schlieren picture of the whole nozzle flow, made up of seven separate images. At the upstream end, the leading and trailing shocks diverge significantly. However, further downstream, where the axial Mach-number gradient is smaller and the boundary layer is thicker, the divergence disappears.

Fig. 4 shows the centerline Mach number distribution obtained from measurements of the intersection angle of the waves, and Fig. 5 is a plot of the flow angle at a number of transverse cuts along the nozzle.

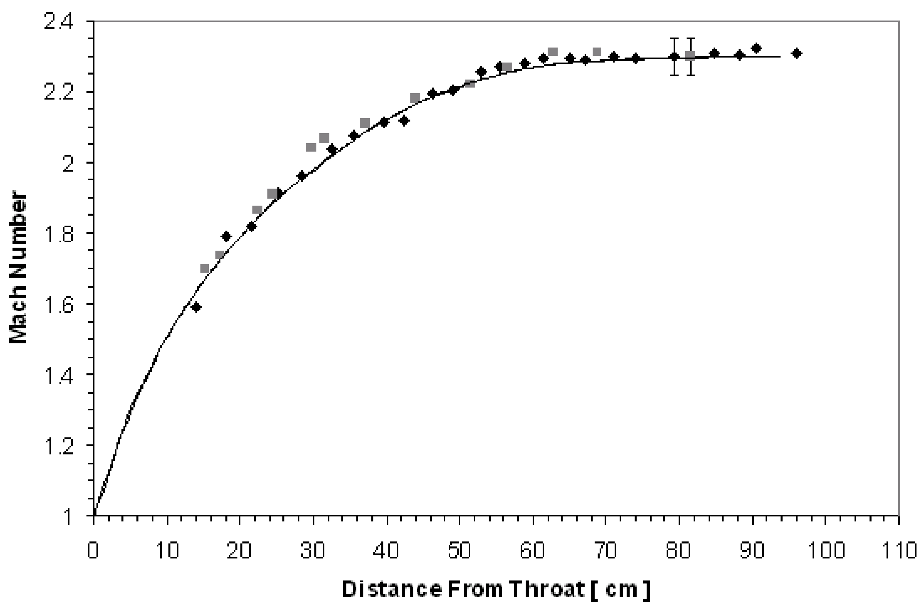

Figure 4. Mach number distribution obtained from measurements of the intersection angles of the waves in Fig 3. The diamonds represent measurements of the intersection of the weak shocks. The squares represent measurements from intersections of the leading characteristics of the trailing expansion waves. The full line is the centerline Mach number distribution obtained by J. J. Korte in his design of this nozzle, which takes account of the boundary layer displacement thickness.

The Mach number distribution was also measured by using the centerline pitot pressure

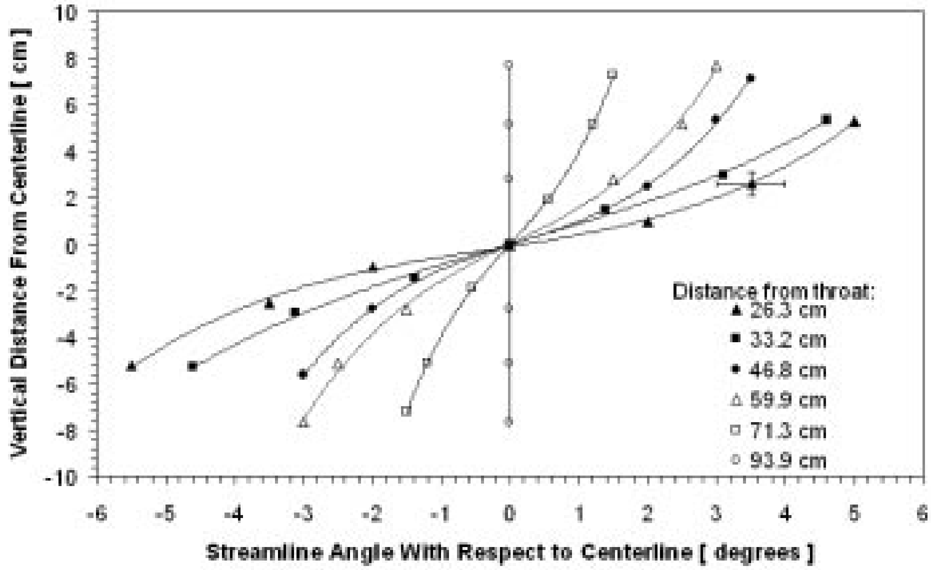

Figure 5. Plot of the streamline angle against transverse distance at several streamwise stations. To within the accuracy of the method, the flow angle at the last station is zero, *i. e.*, the flow in the test section is parallel. The full lines are curves fitted to the experimental points.

distribution and the wall static pressure distribution. However, with the pressure transducers used for this purpose, the accuracy of the Mach wave method was far superior to the pressure technique.

4 Conclusions

The Mach number distribution in the Mach 2.3 nozzle of the new Ludwieg Tube at Caltech has been measured using the Mach wave intersection method. The centerline Mach number distribution agrees very well with the design curve, and the flow in the test section is parallel to within the accuracy of the method. The Mach number in the test section is 2.30±0.03. The Ludwieg Tube principle is very well suited to university research, because of the low cost of operation and the clean flow it provides.

5 References

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Acknowledgments

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